ESTIMATION OF RADIATION LOSS FROM STELLARATORS USING NEOCLASSICAL TRANSPORT THEORY

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Abstract

The radiation power loss from a magnetically confined plasma in a stellarator has been calculated using neoclassical particle and energy fluxes in an energy balance equation. We could obtain a power loss of about 34% of the total input power by numerical calculation. This fraction of the power loss has been found to be in good agreement with the experimentally measured values of 30-40% for our reference machine.

Introduction

The problem of the radiation power loss from a plasma is of great importance for two main reasons: first, it can provide very useful information about the plasma's main parameters; and second, it appears to be one of the major controlling factors in the energy balance problem. In this paper, we will use the energy balance equations for each species (electrons and ions) to calculate, in an appropriate way, the radiated power loss from a reference stellarator. In the following sections, the neoclassical energy and particle fluxes which are used in the energy balance equations will be described in detail, and the radiated power loss will then be computed using some approximations and taking into account a parabolic radial profile and an exponential temporal profile for the species densities and temperatures.

Materials and Methods

Energy Balance Equations

The starting point for our computations will be the following one-dimensional energy balance equations:

$$\frac{3}{2}\frac{\partial}{\partial t}n_eT_e = -\frac{1}{r}\frac{\partial}{\partial r}r\pi_e + q_eE_aS_e + P_e - P_r - P_{ei} \qquad (1)$$

Keywords: Transport theory; Stellarator; Neoclassical fluxes

$$\frac{3}{2}\frac{\partial}{\partial t}n_iT_i = -\frac{1}{r}\frac{\partial}{\partial r}r\pi_i + q_iE_aS_i + P_i + P_{ei} \qquad (2)$$

where n_e, n_i, T_e and T_i are electron and ion densities and temperatures, respectively and r is the radius of the magnetic surface. π_e and S_e are the electron's energy and particle fluxes, respectively which are defined as:

$$\pi_e = \frac{1}{2}n_em_e[(\vec{V_e}\cdot\vec{V_e})\vec{V_e}]$$

$$S_e = n_eV_e$$

π_i and S_i are the corresponding quantities for ions. P_e and P_i are external heating powers for electrons and ions and P_r is the radiation power loss from the plasma. P_{ei} (or P_{ie}) is the amount of heat which is transferred from electrons to ions via collisions (and vice versa).

In order to estimate P_r from the energy balance equations, we consider the sum of the two equations (1) and (2):

$$\frac{3}{2}\frac{\partial}{\partial t}(n_eT_e + n_iT_i) = -\frac{1}{r}\frac{\partial}{\partial r}r(\pi_e + \pi_i) - P_r + P_e \qquad (3)$$

Note that we have taken into account here the ambipolarity conditions:

$q_eS_e = -q_iS_i$. P_i has been taken equal to zero, since we consider only an ECR heating for electrons.

Temporal and Radial Profiles for Temperatures and Densities

As mentioned above, we consider an ECRH heating for our reference stellarator (Heliotion-E), whose main parameters are given in references 1 and 2. The time variation of the electron temperature for the reference stellarator shows an increase of the electron central temperature (r=0) up to about 875 ev. This is reached at a time of about 20 msec after RF source is turned on. Then, the temperature remains constant throughout the heating (RF pulse) period. The same behaviour has been found for the central temperature which shows a saturation at about 20 msec. The attained ion temperature at this period is about 135 ev. Both electrons and ions profiles show a relatively rapid decrease at the end of the RF heating pulse. We could fit the following temporal shapes for the temperatures:

$$T_e(o) = T_{eo}\left(1 - e^{-0.2t}\right)$$
$$T_i(o) = T_{io}\left(1 - e^{-0.4t}\right) \qquad (4)$$

The radial profiles for the species temperatures and densities have been experimentally found to be in the form of (3, 4, 5):

$$T_j(r) = T_j(o)\left(1 - \frac{r^2}{a^2}\right)$$
$$N_j(r) = N_j(o)\left(1 - \frac{r^2}{a^2}\right) \qquad (5)$$

We insert these parabolic profiles into the energy flux formula at RHS of the equation (3) and carry the differentiation with respect to r. π_e and π_i are neoclassically calculated energy fluxes for electrons and ions. It has been shown that for the case of rippled fields in tokamak or stellarator fields, we have to consider the effect of the field inhomogenity (neoclassical transport theory) [3, 4]. In these cases, even a weak field ripple causes an increase of the particle diffusion and conductivity due to the appearance of the locally trapped particles. We will use here the flux formula given in reference [4] by Kovrizhnikh. By a detailed neoclassical treatment of the fluxes, Kovrizhnikh has given the energy fluxes as the sum of π_j^l and π_j^t which correspond respectively to the locally and toroidally trapped particles:

$$\pi_j = \pi_j^l + \pi_j^t$$

where:

$$\pi_e^l = -\frac{3.53\, a_e A_e T_e}{1 + 0.94\,\alpha_e} - \frac{7.4\, a_e B_e T_e}{1 + 0.49\,\alpha_e} \qquad (6)$$

and:

$$\pi_i^l = \pi_i^{HD} - \frac{1.46\, a_i A_i T_i}{1 + 0.39\,\alpha_i} - \frac{3.28\, a_i B_i T_i}{1 + 0.22\,\alpha_i}$$

$$\pi_e^t = -\frac{2.83\, b_e^{(2)} A_e T_e}{1 + 1.86\,\beta_e^{(2)}} - \frac{10.56\, b_e^{(2)} B_e T_e}{1 + 1.16\,\beta_e^{(2)}} \qquad (7)$$

$$\pi_i^t = -\frac{1.90\, b_i^{(2)} A_i T_i}{1 + 0.59\,\beta_i^{(2)}} - \frac{7.19\, b_i^{(2)} B_i T_i}{1 + 0.37\,\beta_i^{(2)}}$$

with:

$$A_j = \frac{N'_j}{N_j} + \frac{q_j \phi'}{T_j} - \frac{3}{2}\frac{T'_j}{T_j} \qquad (j = e, i)$$

and:

$$B_j = \frac{T'_j}{T_j}$$

The primes indicate the derivative with respect to time. The remaining other parameters are defined as:

$$\alpha_j = \frac{\nu_j^* q_s R_o^3 \hat{\Delta}}{r^2 V_j}$$

with:

$$\nu_j^* = \frac{4\sqrt{\pi}}{3}\,\frac{e_j^4 N_j L_j}{m_j^{1/2} T_j^{3/2}}$$

$$a_j = \nu_j^* N_j q_s^2 \hat{\Delta}\left(\frac{R_o V_j}{r\omega_j}\right)^2$$

$$V_j^2 = \frac{T_j}{m}$$

$$b_j^{(1)(2)} = N_j\left(\frac{\nu_j^*}{\omega_j}\right)^{1/2}\frac{r^3\nu_j}{R_o^2}\left[\frac{T_j}{r|e_i|\left(\phi' + E_j^{(1)(2)}\right)}\right]^{3/2}$$

$$\beta_j^{(1)(2)} = 0.1\left[\frac{m_j\omega_j\nu_j^* r}{\in |e_j| E_j^{(1)(2)}}\right]^{3/2} I_1^{-1}$$

$$E_j^{(1)(2)} = \frac{T_j}{|e_j|}\left|\frac{r^3 V_j^2}{\nu_j^*\omega_j R_o^4}\right|^{1/3} K_j^{(1)(2)}$$

$$K_e^{(1)} = 0.12 f_e^{(1)} \quad , \quad K_i^{(1)} = 0.34 f_i^{(1)}$$
$$K_e^{(2)} = 1.31 f_e^{(2)} \quad , \quad K_i^{(2)} = 2.06 f_i^{(2)}$$

$$f_j^{(1)} = \left\{\left[1 + 1.31\frac{N_j T'_j}{N'_j T_j}\right]\frac{r N_j T_j^{-3/2} N'_j}{\int_0^r r dr N_j^2 T_j^{-3/2}}\right\}^{2/3}$$

$$f_j^{(2)} = \left\{\left[1 + 0.45\frac{N'_j T_j}{N_j T'_j}\right]\frac{r N_j^2 T_j^{-3/2} N'_j}{\int_0^r r dr N_j^2 T_j^{-1/2}}\right\}$$

$$I_1 \cong 0.98\, exp\left[-3\alpha_o\left(1 + 0.5\,\alpha_o^3\right) + 0.02/\left(1 + \alpha_o^3\right)\right]$$

$$\alpha_o = \delta / \in q\mu, \in \cong \left| \hat{\in}^2(X) + \left(\frac{n\hat{\in}(x)}{XnN} \right)^2 \right|,$$

$$x = \frac{nNr}{R_o}, \hat{\Delta} = \sqrt{\hat{\in} + \delta} - \sqrt{\hat{\in}} \quad \delta = r/R_o$$

In these formulae, m_j represents the mass, q_j the charge and T_j the temperature, N_j the density and $\omega_j = \frac{q_j B}{m_j C}$, B is the magnetic field on the axis, R is the magnetic field's radius and r is the minor axis of the observation point. The primes indicate the differentiation with respect to r. q_s is the safety factor and $\lambda \approx 114$ is the Coulomb logarithm. $\hat{\in}$ is the relative amplitude of the stellarator field and ϕ is the ambipolar potential. π_i^{HD} is the flux component obtained by hydrodynamical approximations (Kovrizhnikh):

$$\pi_i^{HD} = -2(1 + q_s^2)\, v_i'$$

In deriving the above-mentioned fluxes, it has been supposed that the field has a toroidal harmonics:

$$B = B_o \frac{R_o}{R} \left[1 - \hat{\in}(r) \, Cos\, n(\theta - N\varphi) \right] \quad (8)$$

where

$$\hat{\in} = n \in_o I_n \left(\frac{nNr}{R_o} \right)$$

$\in_o$ is a constant which defines the stellarator field and has the following relationship with the rotational transform:

$$\iota(o) = N(1 - \sqrt{1 - \in_o^2}) \quad (9)$$

Conclusion

Numerical Calculation of the Loss of Power

We consider again equation (3) and replace π_e and π_i by their neoclassical values given by equations (6) and (7).

In accordance with the experimental results given in [6], we fit for the electric field E_a around the center ($r < 10$ cm) the following shape:

$$E_a(r) = ar^2 + br$$

where a and b are some fitting constants given in [7]. Here $E_a(r)$ is given in Gaussian units. Concerning ourselves only with $t > 20$ ms intervals, where the ion and electron temperatures and densities almost lose their time dependences, we could calculate the radiation power loss from:

$$P_r = \frac{1}{r} \frac{\partial}{\partial r} \left[r(\pi_e + \pi_i) \right] - P_e \quad (10)$$

The neoclassical fluxes π_e and π_i will be taken from equations (6) and (7). If we insert the profiles from equation (5) and take for our reference stellarator (10, 3), the radiated power could be calculated. To achieve our calculations, we first have to get, analytically, the following integrals which appear in our equation:

$$I_{ne} = \int_o^r r dr N_e^2 T_e^{-1/2} \quad (11)$$

$$= \int_o^r r dr N_{eo}^2 (1 - r^2/a^2) T_{eo}^{-1/2} (1 - r^2/a^2)^{-1/2}$$

$$\simeq \frac{a^2}{5} N_{eo}^2 T_{eo}^{-1/2} \left[1 - (1 - r^2/a^2)^{5/2} \right]$$

similarly:

$$I_{ni} = \int_o^r r dr N_i^2 T_i^{-1/2} = N_{io}^2 T_{io}^{-1/2} \int_o^r (1 - r^2/a^2)^{3/2} r dr$$

$$= \frac{a^2}{5} T_{io}^{-1/2} N_{io}^2 \left[1 - (1 - r^2/a^2)^{5/2} \right]$$

Now, using the numerical values for the geometrical and magnetic parameters of the reference stellarator in our computer program, we get, finally, a radiated power of about $p_r \simeq 197756$ erg/cm^3 for r = 3 cm. This power loss via radiation is about 34.35% of the external heating power (ECH = 1000 Kw which corresponds to a P_e of 575688 erg/cm^3). This result seems to be in good agreement with the radiation power loss measured by Besshou *et al.*, using a metal bolometer [8]. The same result has been obtained experimentally by one of the authors in CHS (Compact Helical System) at the National Institute for Fusion in Japan.

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